



TIEBREAKER ROUND

Name: _____

Team Name: _____

INSTRUCTIONS

1. You will have 20 minutes to solve 3 problems. Do not begin until instructed to by the proctor.
2. When you would like to submit your answers, please inform your proctor.
3. **Your score will be the number of correct answers, with ties broken by time of submission.**
4. No calculators or electronic devices are allowed.
5. All submitted work must be your own. You may not collaborate with anyone else during the individual round.
6. When time is called, please put your pencil down and hold your paper in the air. **Do not continue to write.** If you continue writing, your score may be disqualified.
7. Do not discuss the problems until all papers have been collected.
8. If you have a question or need to leave the room for any reason, please raise your hand quietly.
9. Good luck!

ACCEPTABLE ANSWERS

1. All answers must be simplified as much as reasonably possible. For example, acceptable answers include $\sin(1^\circ)$, $\sqrt{43}$, or π^2 . Unacceptable answers include $\sin(30^\circ)$, $\sqrt{64}$, or 3^2 .
2. All answers must be exact. For example, π is acceptable, but 3.14 or $22/7$ is not.
3. All rational, non-integer numbers must be expressed in reduced form $\pm \frac{p}{q}$, where p and q are relatively prime positive integers and $q \neq 0$. For example, $\frac{2}{3}$ is acceptable, but $\frac{4}{6}$ is not.
4. All radicals must be fully reduced. For example, $\sqrt{24}$ is not acceptable, and should be written as $2\sqrt{6}$. Additionally, rational expressions cannot contain radicals in the denominator. For example, $\frac{1}{\sqrt{2}}$ is not acceptable, and should be written as $\frac{\sqrt{2}}{2}$.
5. Answers should be expressed in base 10 unless otherwise specified.
6. Complex numbers should be expressed in the form $a + bi$, where both a and b are written in a form compliant with the rules above. In particular, no complex denominators are allowed. For example, $\frac{1+2i}{1-2i}$ should be written as $-\frac{3}{5} + \frac{4}{5}i$ or $\frac{-3+4i}{5}$.
7. If a problem asks for all solutions, you may give the answers in any order. However, no credit will be given if any solution is missing or any solution is given but not correct.
8. Angle measurements should be given in radians unless otherwise specified.
9. Answers must be written legibly to receive credit. Ambiguous answers may be marked incorrect, even if one of the possible interpretations is correct.



TIEBREAKER ROUND

1. What is the minimum number of points in the plane needed such that we can draw an equilateral triangle, a square, a regular hexagon, a regular octagon, and a regular dodecagon (12-sided) using only those points as their vertices?

1. 16

Solution: First, notice that we can embed the triangle, square, and hexagon in the dodecagon. Hence, we only need to consider how to choose points to allow both a regular octagon and regular dodecagon to exist.

We have two possible cases, where the octagon and dodecagon share or do not share a circumcircle. If they do not, two circles intersect at at most two points and hence can share at most two points. This gives us $12 + 8 - 2 = 18$ points. If they do reside in the same circle, they can share at most $\gcd(12, 8) = 4$ points. This construction is possible if we let every second point on the octagon match every third point on the dodecagon. This gives us $12 + 8 - 4 = 16$ points.

Taking the minimum, we get an answer of 16 points.

2. Find the sum of the prime factors of 10981.

2. 218

Solution: $10981 = 10^4 + 10^2 \cdot 3^2 + 3^4 = (10^2 - 10 \cdot 3 + 3^2)(10^2 + 10 \cdot 3 + 3^2) = 79 \cdot 139$ so the answer is $79 + 139 = \boxed{218}$.

An alternative (but more cumbersome) solution: $10981 = 11881 - 900 = 109^2 - 30^2 = (109 + 30)(109 - 30) = 139 \cdot 79$.

3. Let n equal the number of ways to place an integer between 1 and 8, inclusive, in each cell of a grid with 8 rows and 4 columns such that the sum of each row is even and the sum of each column is divisible by 4. How many factors does n have?

3. 82

Solution: We can reduce the problem to counting the number of ways to do it with integers between 1 and 4, then multiplying by $2^{8 \cdot 4}$. Let us first consider arbitrarily filling every cell except for the cells in the last row and last column. There are $4^{7 \cdot 3}$ ways to do this. Now, note that every one of these fillings corresponds to exactly 2^7 ways to fill the whole grid. To see this, note that, because of the row sum condition, there are always exactly 2 choices for each cell in the last column except the last cell, giving 2^7 total choices. After this, every cell in the last row is uniquely determined by the column sum condition. Note that this last row never breaks the even sum condition because the whole grid must have an even sum due to the column sum condition. Therefore, $n = 2^{8 \cdot 4} 4^{7 \cdot 3} 2^7 = 2^{81}$, which has $\boxed{82}$ factors.