WOLVERINE ROUND

Names: ______________________________

Team Name: ______________________________

INSTRUCTIONS

1. Do not begin until instructed to by the proctor.
2. You will have 60 minutes to solve 10 problems.
3. Your score will be the number of correct answers. There is no penalty for guessing or incorrect answers.
4. **Only the official team answers will be graded.** If you are submitting the official answer sheet for your team, indicate this by writing "(OFFICIAL)" next to your team name. Do not submit any unofficial answer sheets.
5. No calculators or electronic devices are allowed.
6. All submitted work must be the work of your own team. You may collaborate with your team members, but no one else.
7. When time is called, please put your pencil down and hold your paper in the air. **Do not continue to write.** If you continue writing, your score may be disqualified.
8. Do not discuss the problems with anyone outside of your team until all papers have been collected.
9. If you have a question or need to leave the room for any reason, please raise your hand quietly.
10. Good luck!

Acceptable Answers

1. All answers must be simplified as much as reasonably possible. For example, acceptable answers include $\sin(1^\circ)$, $\sqrt{43}$, or π^2. Unacceptable answers include $\sin(30^\circ)$, $\sqrt{64}$, or 3^2.

2. All answers must be exact. For example, π is acceptable, but 3.14 or 22/7 is not.

3. All rational, non-integer numbers must be expressed in reduced form $\pm\frac{p}{q}$, where p and q are relatively prime positive integers and $q \neq 0$. For example, $\frac{2}{3}$ is acceptable, but $\frac{4}{6}$ is not.

4. All radicals must be fully reduced. For example, $\sqrt{24}$ is not acceptable, and should be written as $2\sqrt{6}$. Additionally, rational expressions cannot contain radicals in the denominator. For example, $\frac{1}{\sqrt{2}}$ is not acceptable, and should be written as $\frac{\sqrt{2}}{2}$.

5. Answers should be expressed in base 10 unless otherwise specified.

6. Complex numbers should be expressed in the form $a+bi$, where both a and b are written in a form compliant with the rules above. In particular, no complex denominators are allowed. For example, $\frac{1+2i}{1-2i}$ should be written as $-\frac{3}{5}+\frac{4}{5}i$ or $\frac{-3+4i}{5}$.

7. If a problem asks for all solutions, you may give the answers in any order. However, no credit will be given if any solution is missing or any solution is given but not correct.

8. Angle measurements should be given in radians unless otherwise specified.

9. Answers must be written legibly to receive credit. Ambiguous answers may be marked incorrect, even if one of the possible interpretations is correct.

WOLVERINE ROUND

1. (4 points) There are three pairs of white socks and two pairs of black socks in George's drawer. If he picks two socks uniformly at random from his drawer without replacement, what is the probability that the socks are the same color?

1. $\frac{7}{15}$

Solution:

There are $\binom{6}{2}$ ways to pick both white socks and $\binom{4}{2}$ ways to pick both black socks. There are $\binom{10}{2}$ ways to pick the socks.

So the final probability is $\frac{\binom{6}{2}+\binom{4}{2}}{\binom{10}{2}} = \frac{6\cdot 5/2+4\cdot 3/2}{10\cdot 9/2} = \frac{15+6}{45} = \frac{21}{45} = \boxed{\frac{7}{15}}$.

2. (4 points) On an exam, Alice, Bob, and Charlie averaged a score of 6, Alice and Dave averaged a score of 7, and Bob, Charlie, and Dave averaged a score of 8. What was the average score of all four people?

2. 7

Solution:

$$\frac{A+B+C}{3} = 6, \frac{A+D}{2} = 7, \frac{B+C+D}{3} = 8$$
$$\implies A+B+C = 18, A+D = 14, B+C+D = 24$$
$$\implies 2A+2B+2C+2D = 18+14+24 = 56$$
$$\implies \frac{A+B+C+D}{4} = \frac{56}{8} = \boxed{7}$$

3. (Estimation) Submit an expression using all of the following characters exactly once (not including the commas) and no other characters:

$$2,3,4,5,6,7,8,9,+,-,*,/$$

Let P be the value of your expression after evaluating it using standard order of operations. For example, if you submit the expression $457+2*8-96/3$, your value for P will be $457+(2*8)-(96/3) = 441$. Let T be the largest value among all expressions submitted by each team. Then, the number of points you will get for this problem will be $\max(\frac{4P}{T}, 0)$. You will receive 0 points for an expression that is invalid, is illegible, or divides by zero.

3. **N/A**

4. (5 points) If x, y, and z are relatively prime positive integers such that $3x+y-2z = 5x+4y-5z = 0$, what is y?

4. 5

Solution: Using elimination, we get $7x = 3z$ and $5x = 3y$. Then, in order for x, y, and z to be relatively prime, we have $(x, y, z) = (3,5,7)$, so $y = \boxed{5}$.

5. (5 points) Let n and m be positive integers. There are at least 20 positive integers less than n that are co-prime to m. There are at least 25 positive integers less than m that are co-prime to n. Find the least possible value of $n+m$.

5. 52

Solution: It can be shown that the minimum occurs when n and m are the least prime integers greater than 20 and 25, respectively. Then, $n = 23$ and $m = 29$, so $n + m = 52$.

6. (Estimation) Submit a positive integer n. Let M equal the median of all of the positive integers submitted by each team for this problem. Then, your score will be $5 \cdot \min(\frac{n}{M}, \frac{M^2}{n^2})$.

6. **N/A**

7. (6 points) Compute $(\frac{1+i\sqrt{3}}{2})^{2025}$.

7. -1

Solution: $(\frac{1+i\sqrt{3}}{2})^{2025} = (e^{\frac{\pi i}{3}})^{2025} = e^{675\pi i} = e^{\pi i} = \boxed{-1}$.

8. (6 points) Let x_1, x_2, x_3, and x_4 be four non-negative real numbers that sum to 5. What is the maximum possible value of the quantity $(x_1)^1(x_2)^2(x_3)^3(x_4)^4$?

8. 27

Solution: We will use the AM-GM inequality. Rewrite $x_1 + x_2 + x_3 + x_4 = 5$ as
$x_1 + \frac{x_2}{2} + \frac{x_2}{2} + \frac{x_3}{3} + \frac{x_3}{3} + \frac{x_3}{3} + \frac{x_4}{4} + \frac{x_4}{4} + \frac{x_4}{4} + \frac{x_4}{4} = 5$.
By AM-GM, we get

$$\sqrt[10]{(x_1)(\tfrac{x_2}{2})(\tfrac{x_2}{2})(\tfrac{x_3}{3})(\tfrac{x_3}{3})(\tfrac{x_3}{3})(\tfrac{x_4}{4})(\tfrac{x_4}{4})(\tfrac{x_4}{4})(\tfrac{x_4}{4})} \le \frac{x_1 + \frac{x_2}{2} + \frac{x_2}{2} + \frac{x_3}{3} + \frac{x_3}{3} + \frac{x_3}{3} + \frac{x_4}{4} + \frac{x_4}{4} + \frac{x_4}{4} + \frac{x_4}{4}}{10} = \frac{1}{2}$$

$$\implies \sqrt[10]{(x_1)(\tfrac{x_2}{2})^2(\tfrac{x_3}{3})^3(\tfrac{x_4}{4})^4} \le \frac{1}{2}$$

$$\implies (x_1)(\tfrac{x_2}{2})^2(\tfrac{x_3}{3})^3(\tfrac{x_4}{4})^4 \le \frac{1}{2^{10}}$$

$$\implies (x_1)(x_2)^2(x_3)^3(x_4)^4 \le \frac{2^2 3^3 4^4}{2^{10}} = 3^3 = 27.$$

Finally, we check that the equality case, where $x_1 = \frac{x_2}{2} = \frac{x_3}{3} = \frac{x_4}{4}$, is possible. We get that $x_1 = \frac{1}{2}, x_2 = 1, x_3 = \frac{3}{2}$, and $x_4 = 2$, which satisfies $(x_1)(x_2)^2(x_3)^3(x_4)^4 = \boxed{27}$.

9. (Estimation) Circle exactly one of the 5 expressions on your answer slip. The number of points you will receive for this problem will be equal to the expression you chose evaluated at n, where n is the total number of teams that also circled that same expression. For example, if exactly 3 teams circle $\ln(n^{\frac{3}{2}})$, each of the 3 teams that circled $\ln(n^{\frac{3}{2}})$ will get $\ln(3^{\frac{3}{2}})$ points.

$15e^{-\frac{n}{2}}$ $\quad$ $3 + 2\sin(n)$ $\quad$ $\ln(n^{\frac{3}{2}})$ $\quad$ 2.5 $\quad$ $(n \mod 7)$

Solution: Below are the number of teams that circled each expression, along with the number of points received:

$15e^{-\frac{n}{2}}$	$3+2\sin(n)$	$\ln(n^{\frac{3}{2}})$	2.5	$(n \mod 7)$
8 teams, 0.27 pts	4 teams, 1.48 pts	7 teams, 2.91 pts	3 teams, 2.5 pts	7 teams, 0 pts

10. (7 points) Given that $(\log_{2025}(x))^3 + (\log_x(2025))^3 = 52$, what is $\log_{2025}(x) + \log_x(2025)$?

10. 4

Solution: Let $a = \log_{2025}(x) + \log_x(2025)$.

$$\implies a^2 = (\log_{2025}(x) + \log_x(2025))^2 = (\log_{2025}(x))^2 + (\log_x(2025))^2 + 2$$

$$\implies a^3 = ((\log_{2025}(x))^2 + (\log_x(2025))^2 + 2)(\log_{2025}(x) + \log_x(2025)) =$$

$$(\log_{2025}(x))^3 + (\log_x(2025))^3 + 3(\log_{2025}(x) + \log_x(2025)) = 52 + 3a$$

Solving $a^3 - 3a - 52 = 0$, we get $a = \boxed{4}$.

11. (7 points) Given that $(\log_x(45))(\log_y(45)) = 2025$ and $xy = \frac{1}{2025}$, what is $(\log_{45}(x))^2 + (\log_{45}(y))^2$?

11. $\frac{8098}{2025}$

Solution: $(\log_x(45))(\log_y(45)) = 2025 \implies \log_{45}(x)\log_{45}(y) = \frac{1}{2025}$.
$xy = \frac{1}{2025} \implies \log_{45}(xy) = -2 \implies \log_{45}(x)+\log_{45}(y) = -2 \implies (\log_{45}(x))^2+(\log_{45}(y))^2+2\log_{45}(x)\log_{45}(y) = 4 \implies (\log_{45}(x))^2 + (\log_{45}(y))^2 = 4 - \frac{2}{2025} = \boxed{\frac{8098}{2025}}$.

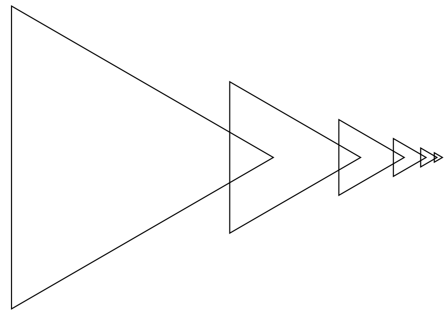
12. (Estimation) Define the function $f(x) = \log_2(2025x)$. Estimate the value of $f(f(f(f(f(1)))))$. The number of points you will receive for this problem is given by the following formula, where P is your estimate, and T is the true value:

$$7 \cdot \min(\frac{P}{T}, \frac{T}{P})$$

12. 14.8784965551

Solution: $\boxed{14.8784965551}$ A rough estimate can be achieved by letting $x = f(f(f(...)))$. We have $x = f(x) = \log_2(2025x)$, which results in $11 + \log_2(x) = x$ after simplification and approximating $\log_2(2025) \approx \log_2(2048) = 11$. We see that x is a bit less than 16 since $11+\log_2(16) = 15 \approx 16$. The value of $f(f(f(f(f(1)))))$ is then a bit less than x, so one may expect the true value to be around 14 or 15.

13. (8 points) Consider the following infinite construction of equilateral triangles, where each triangle's orthocenter is placed at the tip of the previous. If the first triangle has side length 1, and each successive triangle has side length equal to half the side length of the previous triangle, what is the area of the entire construction?



13. $\frac{35\sqrt{3}}{108}$

Solution: We will find the area by finding the sum of the areas of all the triangles and subtracting the areas of the intersections.

The first triangle has area $\frac{1}{2}\cdot 1\cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$. Each triangle afterwards is $1/4$ the area of the previous. So we have:

$$\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{16}+\frac{\sqrt{3}}{64}+\cdots = \frac{\sqrt{3}/4}{1-1/4} = \frac{\sqrt{3}}{3}$$

The area of the first intersection is also an equilateral triangle. It has height $(1/3)\cdot(1/2)\cdot(\sqrt{3}/2) = \sqrt{3}/12$. Then its area is $(\sqrt{3}/12)\cdot(1/12) = \sqrt{3}/144$.

Then each intersection has $1/4$ the area of the previous, so the total area is $\frac{\sqrt{3}/144}{1-1/4} = \frac{\sqrt{3}/36}{3} = \sqrt{3}/108$.

So the area is $\frac{36\sqrt{3}}{108} - \frac{\sqrt{3}}{108} = \boxed{\frac{35\sqrt{3}}{108}}$.

14. (8 points) What is the sum of all of the complex roots with negative real part of the following polynomial:

$$\sum_{n=0}^{11}(-1)^{n+1}x^n$$

14. $-1-\sqrt{3}$

Solution:

$$\sum_{n=0}^{11}(-1)^{n+1}x^n = \frac{x^{12}-1}{x+1}$$

so the roots of the polynomial are the 12^{th} roots of unity excluding -1. The roots with negative real part are $e^{\frac{8\pi i}{12}}, e^{\frac{10\pi i}{12}}, e^{\frac{14\pi i}{12}}$, and $e^{\frac{16\pi i}{12}}$. Using symmetry across the real axis, these roots sum to $2(\cos(\frac{8\pi i}{12})+\cos(\frac{10\pi i}{12})) = 2(-\frac{\sqrt{3}}{2}-\frac{1}{2}) = \boxed{-1-\sqrt{3}}$.

15. (Estimation) Submit an English word present in the Merriam-Webster Dictionary with exactly 5 letters. Assign numerical values to each letter using $A=1, B=2, \ldots, Z=26$. Let n_k be the numerical value assigned to the k^{th}

letter in your word. Let N_k be the mean of the numerical values of the k^{th} letters among all words submitted by each team. Then, the number of points you will receive for this problem will be given by the following formula:

$$\frac{1}{5}\sum_{k=1}^{5}|n_k - N_k|$$

15. **N/A**

16. (9 points) Let $(-\infty, a] \cup (b, \infty)$ be exactly the set of k values such that the equation $\frac{\sin(7x)-\sin(x)}{\cos(6x)+\cos(2x)} = k\sin(x)$ has at least one solution that is not an integer multiple of π. Find $b^2 + ab - a^2$.

16. 11

Solution: Our first observation is that the sums of the arguments of the sine and cosine terms of the numerator and denominator are equal. In this case, it will be convenient to rewrite the expression as a product. To achieve this, we will use a handful of sum and difference trigonometric identities outlined below:

$$\begin{aligned}
\sin(A+B) &= \sin A\cos B + \cos A\sin B \\
\sin(A-B) &= \sin A\cos B - \cos A\sin B \\
\cos(A+B) &= \cos A\cos B - \sin A\sin B \\
\cos(A-B) &= \cos A\cos B + \sin A\sin B
\end{aligned}$$

Now, we have to turn the arguments into the form $(A+B)$ or $(A-B)$ so that when added/subtracted, they leave us a product. In this case, for the numerator, we shall pick $7x = A+B$ and $x = A-B$. For the denominator, we shall pick $6x = A+B$ and $2x = A-B$. This reduces the equation to:

$$\begin{aligned}
\frac{2\cos 4x\sin 3x}{2\cos 4x\cos 2x} &= k\sin x \\
\frac{\sin 3x}{\cos 2x} &= k\sin x
\end{aligned}$$

Using the identities described above, we can obtain:

$$\begin{aligned}
\sin 3x &= 3\sin x - 4\sin^3 x \\
\cos 2x &= 1 - 2\sin^2 x
\end{aligned}$$

This leaves us an equation entirely in sines, which reduces to:

$$\sin x(3 - 4\sin^2 x) = k\sin x(1 - 2\sin^2 x)$$

Since we are looking for solutions that aren't integer multiples of π, we can cancel out $\sin x$ to leave us the following quadratic in sines:

$$k - 3 = (2k-4)\sin^2 x$$

We know the range of $\sin^2 x$ is $[0, 1]$, but we should exclude the zero case as specified by the question. After solving for $0 < \frac{k-3}{2k-4} \leq 1$, we get the set of possible k values to be $(-\infty, 1] \cup (3, \infty)$. Thus, $b^2 + ab - a^2$ is equal to $\boxed{11}$.

17. (9 points) Compute the last three digits of $3^{4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}}}$.

17. 481

Solution: We need to find $3^{4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}}} \ (mod\ 1000)$.

Let us first compute $3^{4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}}} \ (mod\ 8)$ and $3^{4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}}} \ (mod\ 125)$.

$3^2 = 9 = 1$ (mod 8), so $3^{4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}}} = 3^{4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}}} = 1\ (mod\ 8)$.

By Euler's Theorem, $3^{4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}}} = 3^{4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}}\ (mod\ \phi(125))} = 3^{4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}}\ (mod\ 100)}\ (mod\ 125)$.

Therefore, we need to first compute $4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}}\ (mod\ 100)$.

Let us compute $4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}}\ (mod\ 4)$ and $4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}}\ (mod\ 25)$.

$4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}} = 0^{5^{\cdot^{\cdot^{\cdot^{2025}}}}} = 0\ (mod\ 4)$.

$4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}} = 4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}\ (mod\ \phi(25))} = 4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}\ (mod\ 20)} = 4^5 = 24\ (mod\ 25)$.

By the Chinese Remainder Theorem, $4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}} = 24\ (mod\ 100)$.

Then, $3^{4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}}} = 3^{24} = 106\ (mod\ 125)$.

By the Chinese Remainder Theorem, $3^{4^{5^{\cdot^{\cdot^{\cdot^{2025}}}}}} = \boxed{481}\ (mod\ 1000)$.

18. (Estimation) Estimate the total price in cents of today's Jerusalem Garden lunch catering order. The number of points you will receive for this problem is given by the following formula, where P is your estimate, and T is the true value:

$$9 \cdot \min(\frac{p}{T}, \frac{T}{p})$$

18. 164300

19. (10 points) Let $A_1A_2...A_{2026}$ be a convex 2026-sided polygon. Let $B_1, B_2, ..., B_{2025}$ be the midpoints of line segments $A_1A_2, A_2A_3, ..., A_{2025}A_{2026}$, respectively, and let B_{2026} be the midpoint of line segment $A_{2026}A_1$. Let C be a point in the interior of polygon $A_1A_2...A_{2026}$ such that any quadrilateral of the form $CB_nA_{n+1}B_{n+1}$ has area equal to n. What is the area of quadrilateral $CB_{2026}A_1B_1$?

19. 1013

Solution: Notice that since for all n, B_n is the midpoint of A_nA_{n+1}, triangles CA_nB_n and CB_nA_{n+1} have the same area. Then, we see that the sum of the areas of the quadrilaterals for even n equals the sum of the areas of the quadrilaterals for odd n. In other words, the sum of the areas of $CB_1A_2B_2, CB_3A_4B_4, \ldots$, $CB_{2025}A_{2026}B_{2026}$ equals the sum of the areas of $CB_2A_3B_3, CB_4A_5B_5, ...CB_{2026}A_1B_1$. Therefore, the desired area for $CB_{2026}A_1B_1$ is equal to $(1+3+...+2025)-(2+4+...+2024) = \boxed{1013}$.

20. (10 points) How many ordered triplets of positive integers (a, b, c) are there such that $(a^b)^c = 2025^{2025}$?

20. 270

Solution: Notice that a has to be a power of 45. Let $a = 45^n$. Then, $(a^b)^c = ((45^n)^b)^c = 45^{nbc}$, so $45^{nbc} = 2025^{2025} = 45^{4050}$. Then, we have $nbc = 4050$. The prime factorization of 4050 is $2 \cdot 3^4 \cdot 5^2$. Therefore, using stars and bars to distribute the prime factors to n, b, and c, the number of ordered triplets is equal $\binom{1+2}{2}\binom{4+2}{2}\binom{2+2}{2} = 3 \cdot 15 \cdot 6 = \boxed{270}$.

21. (Estimation) Submit an ordered pair (p, q) of integers between 0 and 100, inclusive. Let μ be two-thirds of the average of all such integers p and q submitted by each team. The number of points you will receive for this problem is given by the following formula:

$$10e^{-\frac{1}{2}\cdot(\frac{p-\mu}{10})^2}$$

21. **N/A**

22. (11 points) A particle enters a circular region described by $x^2 + y^2 \le 4$ at a point A and travels along the chord $\sqrt{3}x - y = 1$, finally exiting the region at point B. Let θ equal the measure in radians of $\angle AOB$, where O is the origin. What is $\tan(\theta)$?

22. $-\frac{\sqrt{15}}{7}$

Solution: We first observe that we require $\tan\theta$, which is related to the slope of a line. In this case, computing this will be easier if we have an expression in the slopes of the lines which form the angle θ at the origin. Essentially, we need to look at the equation of the pair of straight lines connecting the origin to the endpoints of the chord.

To achieve this, we will use a technique called "homogenization". Our pair of straight lines must pass through the origin, so the equation must be homogeneous, i.e. it should only have terms of degree 2. We also know it passes through the points of intersection of the chord and the circle. Thus, we can "homogenize" the circle by simply rewriting the radius term using the equation of the chord($\sqrt{3}x - y = 1$).

$$\begin{aligned} x^2 + y^2 &= 4(\sqrt{3}x - y)^2 \\ \implies x^2 + y^2 &= 4(3x^2 + y^2 - 2\sqrt{3}xy) \\ \implies 11x^2 - 8\sqrt{3}xy + 3y^2 &= 0 \end{aligned}$$

Now, let the lines have slopes m_1 and m_2. This equation can thus be rewritten as $(y - m_1x)(y - m_2x) = 0$. Comparing to our equation, we have $m_1m_2 = \frac{11}{3}$, $m_1 + m_2 = \frac{8\sqrt{3}}{3}$. If θ_1 and θ_2 are the inclinations of the lines, θ will be their difference. Thus,

$$\tan\theta = \pm\tan(\theta_1 - \theta_2) = \pm\frac{\tan\theta_1 - tan\theta_2}{1+\tan\theta_1\tan\theta_2}$$

$$\implies \tan\theta = \pm\frac{m_1 - m_2}{1+m_1m_2} = \pm\frac{\sqrt{(m_1+m_2)^2 - 4m_1m_2}}{1+m_1m_2}$$

Using our values of $m_1 + m_2$ and m_1m_2, $\tan\theta = \pm\frac{\sqrt{15}}{7}$. Since in this case θ is obtuse, $\tan\theta$ will take the negative value $\boxed{-\frac{\sqrt{15}}{7}}$.

23. (11 points) Let f, g, and h be continuous invertible functions with domain and range both spanning all real numbers. Suppose that for all real numbers x, we have that $f(g(h(x))) - g(h(x)) - h(x) = 0$. If the graphs of $y = f(x)$ and $y = g(x) + x$ intersect exactly once at the non-origin point (a, b), what is $\frac{b+a}{b}$?

23. $\frac{3}{2}$

Solution: Since domain and ranges are $\mathbb{R}$, we can reduce the equation to $f(g(x)) - g(x) - x = 0$. Reduce the equation once more $f(x) - x - g^{-1}(x) = 0$. Thus, $f(x) - x = g^{-1}(x)$. This means $f(x) - x$ is the inverse of $g(x)$. Setting the two equations for y equal, we get $f(x) - x = g(x)$, which is the intersection of two inverses. If the inverses are defined for all real numbers and only intersect once, they must intersect on $y = x$. This means $g(a) = a$. Plugging back into one formula for y to get (a, b), $b = g(a) + a = 2a$. Thus, $\frac{b+a}{b} = \frac{2a+a}{2a} = \boxed{\frac{3}{2}}$

24. (Estimation) Let P be a regular pentagon in the plane. Choose an edge of P and attach another regular pentagon of the same size to P such that the two share the chosen edge. Iteratively choose random edges on the outside of the resulting figure to attach new regular pentagons to until adding a new pentagon along the randomly chosen edge would overlap the current figure. Let E be the expected number of pentagons in the figure once this process ends (note that this does *not* include the one that would cause the overlap). By submitting an answer A to this question, you will receive points according to the following formula:

$$\max\left(0, 11 - 4|E - A|^{\frac{2}{3}}\right)$$

24. 9.10665