1. (4 points) What is the value of $68^2 - 51^2$?

2. (4 points) Professor DeBacker is on the morning Michigan Math Department walk with x other faculty members and y students. Today, they are visiting the Hyperion Cafe in Kerrytown. He purchases a cup of coffee for each of the faculty members including himself, which costs 3 dollars per cup. Since caffeine is bad for young and developing minds, for the students, he buys cups of hot chocolate, which costs 4 dollars per cup. In total, there are 12 people on the walk including Professor DeBacker, and he spent \$44. What is the value of x?

3. (4 points) In parallelogram ABCD, $AC = AD = 5$ and $AB = 6$. In simplest radical form, what is the length of BD?

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4. (5 points) If $\log_2(1 + 25/2023) = x$ and $\log_2(1 + 1/16) = y$, what is $\log_2(1 + 1/7)$ in terms of x and y?

5. (5 points) Let n be a four-digit base 3 number without leading zeros. If the digits of n sum to 5, what is the difference between the largest and smallest possible values of n?

6. (5 points) In English, the letter "y" is often a vowel but sometimes a consonant. In a long English book, "y" makes up 4% of the vowels, 0.75% of the consonants, and 2% of all the letters. What is the probability that a randomly chosen "y" in the book is a vowel?

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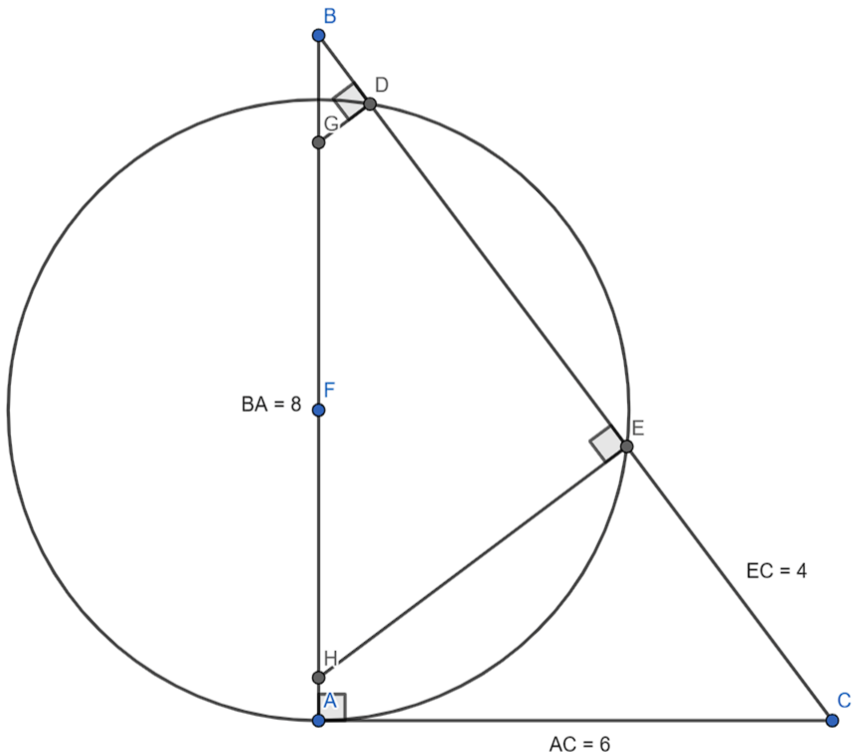
7. (6 points) What is the smallest positive integer k such that both k and $k + 1$ do not divide $12!$?

8. (6 points) Three real numbers a, b, c are selected independently and uniformly at random from $[0, 2]$. What is the probability that $|a + b + c - 3| < 1$?

9. (6 points) What is the sum of all values of x for which $\sin(2x) = \cos(x)$ in the interval $[0, \pi]$?

10. (7 points) If 17 boys and 13 girls are to be randomly seated around a table, what is the expected number of pairs of boys and girls who sit next to each other?

11. (7 points) Professor Koch is building a complex fenced area for her pets, since they have a wide variety of needs. A schematic for the area is shown below.



She plans to store food in a shed represented by a small triangle at the top of this diagram. In order to build a door for this shed, she needs to know its dimensions, starting with the length of its base.

In the above diagram: circle F contains points A, E, and D; $\overline{AC}$ is tangent to circle F at point A; $EC = 4$, $AC = 6$, and $BA = 8$.

What is the length of line segment $\overline{GD}$?

12. (7 points) If $f(x) = x^2 + 12x + 30$, what is the value of $f(f(f(f(59))))$? Your answer may be an expression involving exponents.

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13. (8 points) Suppose positive real numbers x and y satisfy $x + y = 10$. What is the maximum value of x^2y^3?

14. (8 points) If 2022! has X trailing zeros when written in base 2022 and 2023! has Y trailing zeros when written in base 2023, what is the value of $Y - X$?

15. (8 points) Let ABC be a triangle with a right angle at A and incenter I. Suppose the incircle of ABC has radius 1 and is tangent to BC, AC, AB at D, E, F respectively. If the areas of quadrilaterals $AEIF$, $BFID$, and $CDIE$ form a geometric progression in that order, then the area of triangle ABC can be expressed as $p \cdot BD^q$ for rational numbers p and q. In simplest form, what is the ordered pair (p, q)?

16. (9 points) In triangle ABC, $AB = 13, AC = 14$, and $BC = 15$. Let M be a point on BC and $P \neq B, Q \neq C$ be points on AB, AC respectively such that $MB = MC = MP = MQ$. What is the ratio of the area of triangle APQ to the area of quadrilateral $BCPQ$?

17. (9 points) If $\frac{1}{x-1} + \frac{1}{x+1} = \frac{1}{3}$, what is the value of $\frac{1}{x^3-1} + \frac{1}{x^3+1}$?

18. (9 points) If n is a positive integer such that $20n$ has fewer factors than $23n$, what is the smallest possible value of n?

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19. (10 points) Suppose triangle ABC has side lengths $8, 13, 15$. If point P inside the triangle satisfies $\angle APB = \angle BPC = \angle CPA = 120°$, what is the value of $PA + PB + PC$?

20. (10 points) Call two complex numbers $a + bi$ and $c + di$ congruent modulo m if $a - c$ and $b - d$ are both integer multiples of m. Given that $(x, y) = (2, 1)$ is one such ordered pair, for how many ordered pairs of integers (x, y) with $0 \leq x, y \leq 6$ are no two elements of $\{(x + yi)^n | 0 \leq n < 48\}$ congruent modulo 7?

21. (10 points) Estimate the value of $\frac{20^{23}}{23^{20}}$. If your submission is a positive real number S and the true value is T, your score is $10 \cdot \max\left(0, 1 - \max(\log_2(S/T), \log_2(T/S))\right)^3$, rounded to the nearest tenth.

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22. (11 points) Acute triangle ABC with $BC = 20$ and $AB = 19$ is inscribed in a circle with diameter 23. Let D, E, and F be the feet of the perpendiculars from A to BC, B to AC, and C to AB, respectively. Let I be the incenter of DEF. In simplest radical form, what is the length of AI?

23. (11 points) There are 100 pebbles placed in a vertical column. At each step you can pick the top pebble of one column that has at least two more pebbles than the column immediately to the right and move that pebble to the column immediately to the right (if there is no column of pebble to the right, consider it to be a column of 0 pebbles). Eventually, this process must stop. If m and M are the minimum and maximum possible number of steps it takes for this process to stop, what is the ordered pair (m, M)?

24. (11 points) Pick a point (x, y, z), where x, y, z are real numbers satisfying $0 \leq x, y, z \leq 10$. If you enter a valid submission, your score on this problem will be $(x + y + z)$ multiplied by the distance between your point and the closest point submitted by another team, scaled so that the maximum score among all teams is 11 points.