



## TEAM ROUND

Names: \_\_\_\_\_

Team Name: \_\_\_\_\_

## INSTRUCTIONS

1. Do not begin until instructed to by the proctor.
2. You will have 60 minutes to solve 10 problems.
3. Your score will be the number of correct answers. There is no penalty for guessing or incorrect answers.
4. **Only the official team answers will be graded.** If you are submitting the official answer sheet for your team, indicate this by writing “(OFFICIAL)” next to your team name. Do not submit any unofficial answer sheets.
5. No calculators or electronic devices are allowed.
6. All submitted work must be the work of your own team. You may collaborate with your team members, but no one else.
7. When time is called, please put your pencil down and hold your paper in the air. **Do not continue to write.** If you continue writing, your score may be disqualified.
8. Do not discuss the problems with anyone outside of your team until all papers have been collected.
9. If you have a question or need to leave the room for any reason, please raise your hand quietly.
10. Good luck!



## ACCEPTABLE ANSWERS

1. All answers must be simplified as much as reasonably possible. For example, acceptable answers include  $\sin(1^\circ)$ ,  $\sqrt{43}$ , or  $\pi^2$ . Unacceptable answers include  $\sin(30^\circ)$ ,  $\sqrt{64}$ , or  $3^2$ .
2. All answers must be exact. For example,  $\pi$  is acceptable, but 3.14 or  $22/7$  is not.
3. All rational, non-integer numbers must be expressed in reduced form  $\pm\frac{p}{q}$ , where  $p$  and  $q$  are relatively prime positive integers and  $q \neq 0$ . For example,  $\frac{2}{3}$  is acceptable, but  $\frac{4}{6}$  is not.
4. All radicals must be fully reduced. For example,  $\sqrt{24}$  is not acceptable, and should be written as  $2\sqrt{6}$ . Additionally, rational expressions cannot contain radicals in the denominator. For example,  $\frac{1}{\sqrt{2}}$  is not acceptable, and should be written as  $\frac{\sqrt{2}}{2}$ .
5. Answers should be expressed in base 10 unless otherwise specified.
6. Complex numbers should be expressed in the form  $a + bi$ , where both  $a$  and  $b$  are written in a form compliant with the rules above. In particular, no complex denominators are allowed. For example,  $\frac{1+2i}{1-2i}$  should be written as  $-\frac{3}{5} + \frac{4}{5}i$  or  $\frac{-3+4i}{5}$ .
7. If a problem asks for all solutions, you may give the answers in any order. However, no credit will be given if any solution is missing or any solution is given but not correct.
8. Angle measurements should be given in radians unless otherwise specified.
9. Answers must be written legibly to receive credit. Ambiguous answers may be marked incorrect, even if one of the possible interpretations is correct.



## TEAM ROUND

1. In the 2023 Michigan Math Meet, a competition held at the University of Michigan in March 2023, some number of teams and 23 students not on any team participated. Competition organizers confirmed that each team had exactly 5 students and no student was on more than one team. Then, when they were scoring the individual round, they observed that every student received an integer score, yet the median score on the individual round was a 3.5. If there were over 100 participants and exactly one third of them were from the city of Treetown, what is the smallest number of teams that could have been present at the competition?  
1. \_\_\_\_\_
2. A card is randomly drawn from a standard deck of cards. After recording its value, the card is put back into the deck and the deck is reshuffled. This process is repeated three times. Supposing that aces are counted as 1, and jacks, queens, and kings are all counted as 10, what is the probability that the sum of the numbers on the three cards drawn add up to exactly 20?  
2. \_\_\_\_\_
3. An equilateral triangle and a regular hexagon each have an area of 2023. In simplest radical form, what is the ratio of the perimeter of the hexagon to the perimeter of the triangle?  
3. \_\_\_\_\_
4. There is a 9 by 9 grid with each cell colored maize or blue. A row is called majority-blue if and only if it has at least five blue cells. Given only that the grid has  $n$  blue cells in total, the largest and smallest possible number of majority-blue rows differ by 7. What is the sum of all possible values of  $n$ ?  
4. \_\_\_\_\_
5. Partition the ten smallest positive integers into sets  $S$  and  $T$  such that the product of the elements of  $S$  is an integer multiple of the product of the elements of  $T$ , and the sum of the elements of  $S$  is minimized. What is the average of the elements in  $T$ ?  
5. \_\_\_\_\_
6. Suppose  $P(x) = x^2 - 6x + a$ , where  $a$  is a real number. What is the maximum possible value of  $a$  such that the graphs of  $P(x)$  and  $P(P(x))$  intersect?  
6. \_\_\_\_\_
7. There are 10 coins, numbered 1 through 10, where the coin numbered  $k$  has a  $\frac{k+1}{k+2}$  chance of landing on heads and  $\frac{1}{k+2}$  chance of landing on tails. If all 10 coins are flipped, what is the probability that there are an even number of heads?  
7. \_\_\_\_\_
8. Let ABCD be a square with area 100 and E, F, G, H be points on AB, BC, CD, DA respectively such that EFGH is a square with area 75. Let I, J, K, L be the intersections of AG and BH, BH and CE, CE and DF, DF and AG, respectively. What is the sum of all possible areas of quadrilateral IJKL?  
8. \_\_\_\_\_



9. Given that the polynomial  $P(x) = x^4 - 7x^3 - 59x^2 + 37x + 23$  has four real roots between -6 and 12, let  $Q(x)$  be the degree 6 monic polynomial with 6 roots real roots, each of which is the sum of two distinct roots of  $P(x)$ . Find the sum of squares of all real numbers  $t$  such that  $Q(t) = Q(100)$ .

9. \_\_\_\_\_

10. Given that  $\frac{7^5 - 1}{6} = 2801$  is prime, find the smallest positive integer  $n$  such that 2801 divides  $n^2 + n - 1$ .

10. \_\_\_\_\_