



INDIVIDUAL ROUND

Name: _____

Team Name: _____

INSTRUCTIONS

1. Do not begin until instructed to by the proctor.
2. You will have 60 minutes to solve 10 problems.
3. Your score will be the number of correct answers. There is no penalty for guessing or incorrect answers.
4. No calculators or electronic devices are allowed.
5. All submitted work must be your own. You may not collaborate with anyone else during the individual round.
6. When time is called, please put your pencil down and hold your paper in the air. **Do not continue to write.** If you continue writing, your score may be disqualified.
7. Do not discuss the problems until all papers have been collected.
8. If you have a question or need to leave the room for any reason, please raise your hand quietly.
9. Good luck!



ACCEPTABLE ANSWERS

1. All answers must be simplified as much as reasonably possible. For example, acceptable answers include $\sin(1^\circ)$, $\sqrt{43}$, or π^2 . Unacceptable answers include $\sin(30^\circ)$, $\sqrt{64}$, or 3^2 .
2. All answers must be exact. For example, π is acceptable, but 3.14 or $22/7$ is not.
3. All rational, non-integer numbers must be expressed in reduced form $\pm\frac{p}{q}$, where p and q are relatively prime positive integers and $q \neq 0$. For example, $\frac{2}{3}$ is acceptable, but $\frac{4}{6}$ is not.
4. All radicals must be fully reduced. For example, $\sqrt{24}$ is not acceptable, and should be written as $2\sqrt{6}$. Additionally, rational expressions cannot contain radicals in the denominator. For example, $\frac{1}{\sqrt{2}}$ is not acceptable, and should be written as $\frac{\sqrt{2}}{2}$.
5. Answers should be expressed in base 10 unless otherwise specified.
6. Complex numbers should be expressed in the form $a + bi$, where both a and b are written in a form compliant with the rules above. In particular, no complex denominators are allowed. For example, $\frac{1+2i}{1-2i}$ should be written as $-\frac{3}{5} + \frac{4}{5}i$ or $\frac{-3+4i}{5}$.
7. If a problem asks for all solutions, you may give the answers in any order. However, no credit will be given if any solution is missing or any solution is given but not correct.
8. Angle measurements should be given in radians unless otherwise specified.
9. Answers must be written legibly to receive credit. Ambiguous answers may be marked incorrect, even if one of the possible interpretations is correct.

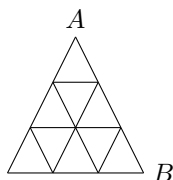


INDIVIDUAL ROUND

1. Two unit circles are drawn so that their centers are located $\sqrt{3}$ units apart. Find the area of the intersection between the two circles.

1. _____

2. Consider the large equilateral triangle below that has been divided into 9 smaller equilateral triangles. How many ways can you walk from point A to point B if you can only go straight or turn in 120-degree angles at intersections and moving in the upwards direction is not allowed?



2. _____

3. If $16^{16^{16}} = 4^{4^x}$, what is the value of x ?

3. _____

4. What is the least positive integer n such that 2023 is equal to the sum of n perfect squares, i.e. there exist a_1, a_2, \dots, a_n such that $2023 = a_1^2 + a_2^2 + \dots + a_n^2$?

4. _____

5. Martha rolls four 6-sided fair dice. What is the probability that the numbers which come up cannot be the side lengths of a quadrilateral?

5. _____

6. If $20 \sin x + 23 \cos x = 27$, what is the largest possible value of $23 \sin x - 20 \cos x$?

6. _____

7. What is the distance between the incenter and the centroid of a triangle whose side lengths are 2021, 2022, 2023?

7. _____

8. How many functions $f : \{1, 2, \dots, 6\} \rightarrow \{1, 2, \dots, 6\}$ are there such that $f^k(k) = k$ for all $k = 1, 2, \dots, 6$, where f^k denotes a composition of f with itself k times?

8. _____

9. Consider the number $2^{2^{\dots^{2^2}}}$ where there are a total of 2023 2's. What is the remainder when we divide that number by 2023?

9. _____

10. Let a, b, c be the roots of $x^3 = 20 + 2x + 3x^2$, and let $p = a^2b + a^2c, q = b^2a + b^2c, r = c^2a + c^2b$. What is the value of $p + q + r - pqr$?

10. _____